Maths Expertes **Corrigé du DS2** **Novembre 2025**

Exercice I

1) $\underline{Re(z) = 1 \ ; \ Im(z) = -4}$

$\underline{Re(z'') = 0 \ ; \ Im(z'') = 8}$

2) $\boxed{z_1} = 13 - 4i - (-7 - 5i) + 3i = 13 - 4i + 7 + 5i + 3i = \boxed{20 + 4i}$

$z_2 = (2 - i)(1 - 5i) - 2(3 - 4i)$

$\boxed{z_2} = 2 - 10i - i + 5i^2 - 6 + 8i = 2 - 5 - 6 - 3i = \boxed{-9 - 3i}$

$\boxed{z_3} = (\sqrt{2} - 2\sqrt{3}i)(\sqrt{2} + 2\sqrt{3}i) = \sqrt{2}^2 - (2\sqrt{3}i)^2 = 2 - 12i^2 = \boxed{14}$

$\boxed{z_4} = 1 + i + i^2 + i^3 + i^4 = 1 + i - 1 - i + 1 = \boxed{1}$

$z_5 = \left(1 - \frac{2}{3}i\right)^2 + (2 - \sqrt{3}i)^2$

$z_5 = 1 - \frac{4}{3}i + \left(\frac{2}{3}i\right)^2 + 4 - 4\sqrt{3}i + (\sqrt{3}i)^2$

$z_5 = 1 - \frac{4}{3}i + \frac{4}{9}i^2 + 4 - 4\sqrt{3}i + 3i^2$

$z_5 = 1 - \frac{4}{9} + 4 - 3 - \frac{4}{3}i - 4\sqrt{3}i$

$\boxed{z_5 = \frac{14}{9} - \left(\frac{4 + 12\sqrt{3}}{3}\right)i}$

Exercice II

1) $Z = \dfrac{-2+3i}{1-5i} = \dfrac{(-2+3i)(1+5i)}{(1-5i)(1+5i)} = \dfrac{-2-10i+3i+15i^2}{1^2+5^2} = \dfrac{-17-7i}{26}$

$\boxed{Z = \dfrac{-17}{26} - \dfrac{7}{26}i}$

2) $2iz+1-i = z+(3+2i)^2$

$2iz - z = -1+i+(3+2i)^2$

$z(-1+2i) = -1+i+9+12i-4$

$z(-1+2i) = 4+13i$

$z = \dfrac{4+13i}{-1+2i} = \dfrac{(4+13i)(-1-2i)}{(-1+2i)(-1-2i)} = \dfrac{-4-8i-13i-26i^2}{(-1)^2+2^2}$

$z = \dfrac{22-21i}{5} = \dfrac{22}{5} - \dfrac{21}{5}i$ $\quad \boxed{\mathcal{S}_{\mathbb{C}} = \left\{\dfrac{22}{5} - \dfrac{21}{5}i\right\}}$.

3) $z^2+2z+9 = 5 \iff z^2+2z+4=0$ (de la forme : $az^2+bz+c=0$ avec : $a=1$; $b=2$; $c=4$).

$\Delta = b^2-4ac = 2^2-4\times1\times4 = 4-16 = -12$

$\Delta<0$, donc l'équation a deux racines dans $\mathbb{C}$:

$$\begin{cases} z_1 = \dfrac{-b-\sqrt{|\Delta|}\,i}{2a} = \dfrac{-2-\sqrt{|-12|}\,i}{2} = \dfrac{-2-\sqrt{12}\,i}{2} = \dfrac{-2-2\sqrt{3}i}{2} = -1-\sqrt{3}i \\ z_2 = \overline{z_1} = -1+\sqrt{3}i \end{cases}$$

$\boxed{\mathcal{S}_{\mathbb{C}} = \{-1-\sqrt{3}i\ ;\ -1+\sqrt{3}i\}}$

4) $z = a+bi$ avec a et b réels.

$z^2 = i \iff (a+bi)^2 = i \iff a^2+2abi-b^2 = i \iff a^2-b^2+(2ab-1)i=0$

$z^2 = i \iff \begin{cases} a^2-b^2=0 \\ 2ab-1=0 \end{cases}$ (car un nb complexe est nul si et seulement si sa partie réelle et sa partie imaginaire sont nulles).

$z^2 = i \iff \begin{cases} (a-b)(a+b)=0 \\ 2ab=1 \end{cases} \iff \begin{cases} a-b=0 \\ 2ab=1 \end{cases}$ ou $\begin{cases} a+b=0 \\ 2ab=1 \end{cases}$

$z^2 = i \iff \begin{cases} a=b \\ 2a^2=1 \end{cases}$ ou $\begin{cases} b=-a \\ -2a^2=1 \end{cases} \to$ Pas de solution car $a\in\mathbb{R}$ donc $a^2\geq0$ donc $-2a^2\leq0$

$z^2 = i \iff \begin{cases} a^2=\frac{1}{2} \\ a=b \end{cases} \iff \left| \begin{array}{l} a=b=\sqrt{\frac{1}{2}}=\frac{\sqrt{2}}{2} \\ \text{ou} \\ a=b=-\sqrt{\frac{1}{2}}=-\frac{\sqrt{2}}{2}. \end{array} \right. \iff \left| \begin{array}{l} z=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i \\ \text{ou} \\ z=-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i \end{array} \right.$

$\boxed{\mathcal{S}_{\mathbb{C}} = \left\{-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\ ;\ \frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right\}}$

Exercice III

Montrons que $Z = \overline{Z}$: Par caractérisation du cours, on aura $Z \in \mathbb{R}$.

Or $Z = \dfrac{u+v}{1+uv}$, donc $\overline{Z} = \overline{\left(\dfrac{u+v}{1+uv}\right)} = \dfrac{\overline{u+v}}{\overline{1+uv}} = \dfrac{\overline{u}+\overline{v}}{1+\overline{u}\times\overline{v}}$ par propriétés du conjugué).

Or $u\overline{u} = 1$, donc $\overline{u} = \dfrac{1}{u}$ et $v\overline{v} = 1$, donc $\overline{v} = \dfrac{1}{v}$.

Donc $$\overline{Z} = \frac{\frac{1}{u}+\frac{1}{v}}{1+\frac{1}{u}\times\frac{1}{v}} = \frac{\frac{v}{uv}+\frac{u}{uv}}{1+\frac{1}{uv}} = \frac{\frac{u+v}{uv}}{\frac{uv+1}{uv}} = \frac{u+v}{uv}\times\frac{uv}{uv+1}$$

$\underline{\underline{\overline{Z}}} = \dfrac{u+v}{uv+1} = \underline{\underline{Z}}$. Donc $\underline{Z \in \mathbb{R}}$.