Première Maths Groupe 4 **Corrigé du DS7** **Février 2023**

Exercice I

1)

°	180	140	y
rad	π	x	$\frac{3\pi}{20}$

$x = \frac{140\pi}{180} = \frac{7\pi}{9}$ et $y = \frac{180 \times \frac{3\pi}{20}}{\pi} = 27$

donc $\boxed{140° = \frac{7\pi}{9} \text{ rad}}$ et $\boxed{\frac{3\pi}{20} \text{ rad} = 27°}$.

2) $\boxed{A} = \sin\left(\frac{5\pi}{4}\right) = \sin\left(\pi + \frac{\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = \boxed{-\frac{\sqrt{2}}{2}}$.

$\boxed{B} = \cos\left(-\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \boxed{\frac{1}{2}}$

2) $C = \left(\cos(x+4\pi)\right)^2 + \left(\sin(\pi - x)\right)^2$

$\boxed{C} = \left(\cos(x)\right)^2 + \left(\sin(x)\right)^2 = \boxed{1}$

$D = \left(\cos(x) + \sin(y)\right)^2 + \left(\cos(x+2\pi) + \sin(-y)\right)^2$

$D = \left(\cos^2(x) + 2\cos(x)\sin(y) + \sin^2(y)\right) + \left(\cos(x) - \sin(y)\right)^2$

$D = \cos^2(x) + \cancel{2\cos(x)\sin(y)} + \sin^2(y) + \cos^2(x) - \cancel{2\cos(x)\sin(y)} + \sin^2(y)$

$\boxed{D} = 2\cos^2(x) + 2\sin^2(y) = \boxed{2\left(\cos^2(x) + \sin^2(y)\right)}$.

3) $-\pi < x < -\frac{\pi}{2}$ et $\cos(x) = -\frac{2}{3}$.

Or $\cos^2(x) + \sin^2(x) = 1$, donc : $\left(-\frac{2}{3}\right)^2 + \sin^2(x) = 1$

$\sin^2(x) = 1 - \left(-\frac{2}{3}\right)^2 = 1 - \frac{4}{9} = \frac{5}{9}$ et $\sin(x) \in \left\{-\sqrt{\frac{5}{9}} ; \sqrt{\frac{5}{9}}\right\}$

Or $-\pi < x < -\frac{\pi}{2}$, donc $\sin(x) < 0$, donc $\boxed{\sin(x)} = -\sqrt{\frac{5}{9}} = \boxed{-\frac{\sqrt{5}}{3}}$

Exercice 2

a) $2\cos(x) - \sqrt{3} = 0 \iff \cos(x) = \frac{\sqrt{3}}{2} = \cos\left(\frac{\pi}{6}\right) \iff \begin{cases} x = \frac{\pi}{6} + 2k\pi \\ \text{ou} \\ x = -\frac{\pi}{6} + 2k\pi \end{cases}$ $k \in \mathbb{Z}$.

$\boxed{S_{\mathbb{R}} = \left\{-\frac{\pi}{6} + 2k\pi, k \in \mathbb{Z}\right\} \cup \left\{\frac{\pi}{6} + 2k\pi ; k \in \mathbb{Z}\right\}}$

$\boxed{S_{]-\pi;\pi]} = \left\{-\frac{\pi}{6} ; \frac{\pi}{6}\right\}.}$

b) $\sin\left(2x+\frac{\pi}{3}\right)=1=\sin\left(\frac{\pi}{2}\right) \iff \begin{cases} 2x+\frac{\pi}{3}=\frac{\pi}{2}+2k\pi \\ \text{ou} \\ 2x+\frac{\pi}{3}=\pi-\frac{\pi}{2}+2k\pi \end{cases}$ même équation.

$\iff x=\frac{1}{2}\left(\frac{\pi}{2}-\frac{\pi}{3}+2k\pi\right)$

$x=\frac{1}{2}\left(\frac{3\pi}{6}-\frac{2\pi}{6}+2k\pi\right)=\frac{1}{2}\left(\frac{\pi}{6}+2k\pi\right)=\frac{\pi}{12}+k\pi$

$\boxed{S_{\mathbb{R}}=\left\{\frac{\pi}{12}+k\pi\ ;\ k\in\mathbb{Z}\right\}}$ $\boxed{S_{]-\pi;\pi]}=\left\{-\frac{11\pi}{12}\ ;\ \frac{\pi}{12}\right\}}$

Exercice III

$f(x)=\dfrac{\sin(x)}{\cos(x)}$.

1a) $\cos(x)=0$.

$\cos(x)=0 \iff \cos(x)=\cos\left(\frac{\pi}{2}\right) \iff \begin{cases} x=\frac{\pi}{2}+2k\pi \\ \text{ou} \\ x=-\frac{\pi}{2}+2k\pi \end{cases}, k\in\mathbb{Z}$. $S=\left\{\frac{\pi}{2}+2k\pi, k\in\mathbb{Z}\right\} \cup \left\{-\frac{\pi}{2}+2k\pi\ ;\ k\in\mathbb{Z}\right\}$

$\mathcal{D}_f=\mathbb{R}-S=\mathbb{R}-\left\{\left\{\frac{\pi}{2}+2k\pi, k\in\mathbb{Z}\right\}\cup\left\{-\frac{\pi}{2}+2k\pi\ ;\ k\in\mathbb{Z}\right\}\right\}$.

1b) Pour tout réel $x\in\mathcal{D}_f$: $\boxed{f(-x)}=\dfrac{\sin(-x)}{\cos(-x)}=\dfrac{-\sin(x)}{\cos(x)}=\boxed{-f(x)}$

1c) Pour tout réel $x\in\mathcal{D}_f$: $\boxed{f(x+\pi)}=\dfrac{\sin(x+\pi)}{\cos(x+\pi)}=\dfrac{-\sin(x)}{-\cos(x)}=\boxed{f(x)}$

1d) $\boxed{f(0)}=\dfrac{\sin(0)}{\cos(0)}=\dfrac{0}{1}=\boxed{0}$

$\boxed{f\left(\frac{\pi}{6}\right)}=\dfrac{\sin\left(\frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{6}\right)}=\dfrac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}=\frac{1}{2}\times\frac{2}{\sqrt{3}}=\frac{1}{\sqrt{3}}=\boxed{\frac{\sqrt{3}}{3}}$

$\boxed{f\left(\frac{\pi}{4}\right)}=\dfrac{\sin\left(\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}\right)}=\dfrac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}=\boxed{1}$

$\boxed{f\left(\frac{\pi}{3}\right)}=\dfrac{\sin\left(\frac{\pi}{3}\right)}{\cos\left(\frac{\pi}{3}\right)}=\dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}=\frac{\sqrt{3}}{2}\times 2=\boxed{\sqrt{3}}$

1e) $\mathcal{C}_f$ coupe l'axe des abscisses aux solutions de l'équation : $f(x)=0$.

$f(x)=0 \iff \dfrac{\sin(x)}{\cos(x)}=0 \iff \sin(x)=0 \iff x=k\pi, k\in\mathbb{Z}$.

$\mathcal{C}_f$ coupe l'axe des x en $A_k(k\pi\ ;0)$ où $k\in\mathbb{Z}$.

2a) $f(x) = \dfrac{\sin(x)}{\cos(x)} = \dfrac{u(x)}{v(x)}$ avec : $\begin{cases} u(x) = \sin(x) \\ u'(x) = \cos(x) \end{cases}$ $\begin{cases} v(x) = \cos(x) \\ v'(x) = -\sin(x) \end{cases}$.

$$f'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{v^2(x)} = \frac{\cos(x) \times \cos(x) - \sin(x) \times (-\sin(x))}{(\cos(x))^2}$$

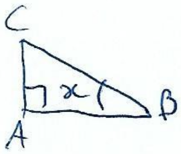
$$\boxed{f'(x)} = \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \boxed{\frac{1}{\cos^2(x)}}$$

2b) Pour tout $x \in \left]-\frac{\pi}{2} ; \frac{\pi}{2}\right[$, $\cos^2(x) > 0$ et $1 > 0$, donc $\dfrac{1}{\cos^2(x)} > 0$ et $f'(x) > 0$

donc $\boxed{f \text{ est strictement croissante sur } \left]-\frac{\pi}{2} ; \frac{\pi}{2}\right[}$.

2c) T a pour équation réduite : $y = f'(0)(x-0) + f(0)$ avec : $\begin{cases} f'(0) = \dfrac{1}{\cos^2(0)} = \dfrac{1}{1^2} = 1 \\ f(0) = 0 \text{ (cf. 2a)}. \end{cases}$

$\boxed{y = x}$

3) 

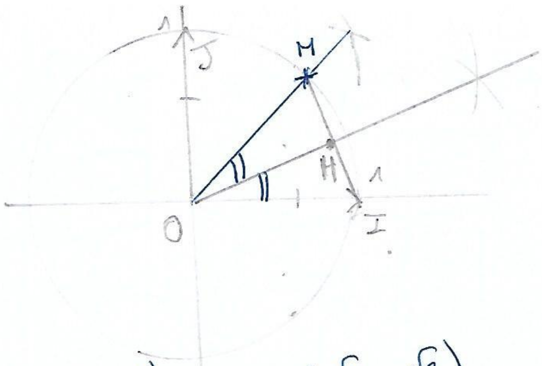
$\cos(x) = \dfrac{AB}{BC}$; $\sin(x) = \dfrac{AC}{BC}$

donc $\dfrac{\sin(x)}{\cos(x)} = \dfrac{\frac{AC}{BC}}{\frac{AB}{BC}} = \dfrac{AC}{BC} \times \dfrac{BC}{AB} = \dfrac{AC}{AB} = \dfrac{\text{côté opposé}}{\text{côté adjacent}}$.

donc $\boxed{f(x) = \tan(x)}$

Exercice IV

1)
3a)



2a) $M\left(\cos\left(\frac{\pi}{4}\right) ; \sin\left(\frac{\pi}{4}\right)\right)$, donc $M\left(\frac{\sqrt{2}}{2} ; \frac{\sqrt{2}}{2}\right)$.

2b) $I(1;0)$ et $M\left(\frac{\sqrt{2}}{2} ; \frac{\sqrt{2}}{2}\right)$, donc $\overrightarrow{IM}\begin{pmatrix} \frac{\sqrt{2}}{2}-1 \\ \frac{\sqrt{2}}{2} \end{pmatrix}$

donc $IM = \sqrt{\left(\frac{\sqrt{2}}{2}-1\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2} = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 - 2\times\frac{\sqrt{2}}{2} + 1 + \left(\frac{\sqrt{2}}{2}\right)^2}$

$\boxed{IM} = \sqrt{\frac{1}{2} - \sqrt{2} + \frac{1}{2} + 1} = \boxed{\sqrt{2-\sqrt{2}}}$

3b) $\widehat{IOH} = \frac{\frac{\pi}{4}}{2} = \frac{\pi}{8}$ car (OH) est la bissectrice de $\widehat{IOM}$.

Dans le triangle OIH rectangle en H, on a : $OI = 1$ et $\widehat{IOH} = \frac{\pi}{8}$.

$\sin(\widehat{IOH}) = \frac{IH}{OI}$ donc $\sin\left(\frac{\pi}{8}\right) = \frac{IH}{1}$ donc $IH = \sin\left(\frac{\pi}{8}\right)$.

Par suite, comme H est le milieu de $[IM]$, on a : $\boxed{IM = 2\sin\left(\frac{\pi}{8}\right)}$

3c) $IM = \sqrt{2-\sqrt{2}}$ (q. 2b) et $IM = 2\sin\left(\frac{\pi}{8}\right)$ (q. 3b).

donc : $\underline{2\sin\left(\frac{\pi}{8}\right)} = \underline{\sqrt{2-\sqrt{2}}}$, et $\boxed{\sin\left(\frac{\pi}{8}\right) = \frac{\sqrt{2-\sqrt{2}}}{2}}$